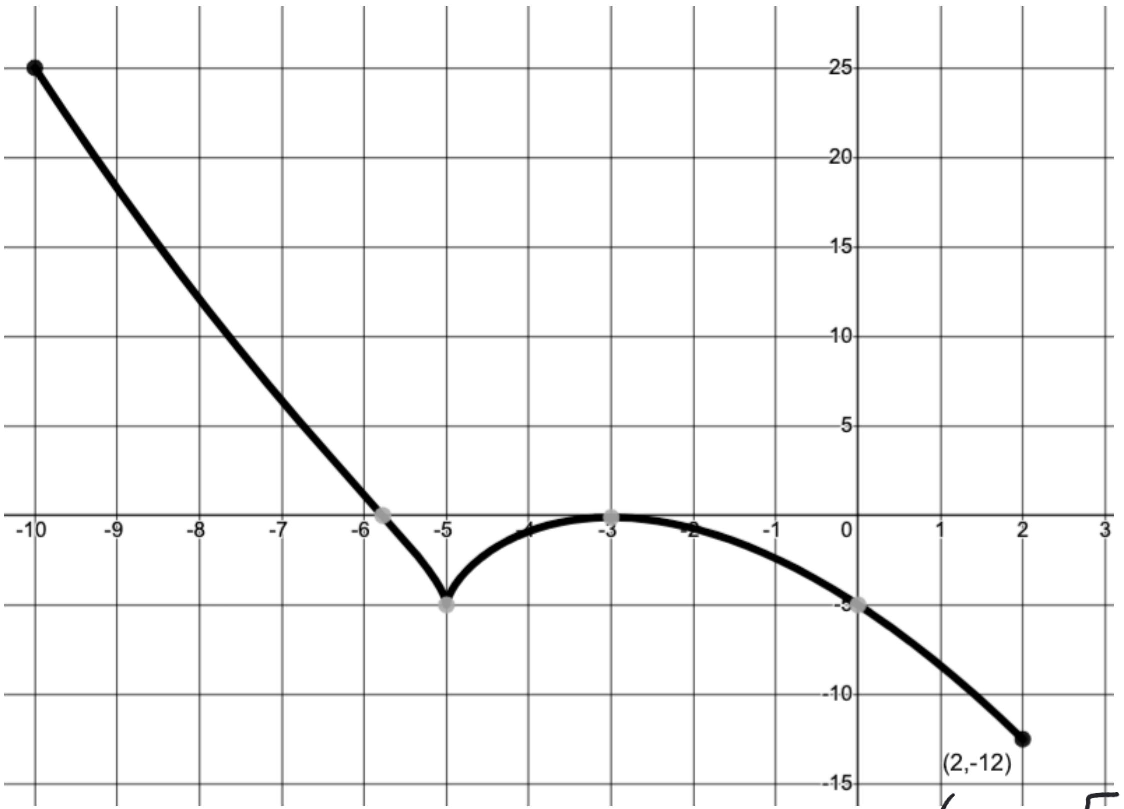
Q7 3.1/3.7i

(1) In the following graph, approximate the following: (1 point each)



a) *Coordinates* for any local minimums: (-5,-5)

b) *Coordinates* for any local maximums: (-3,0)

c) *Value* of absolute minimum (if any): -12

d) *Value* of absolute maximum (if any): 25

e) *x coordinate that yields* absolute maximum (if any): -10

(2) True or False:

a) Local extrema can only occur at critical numbers. True

b) Everywhere there is an absolute extremum there must also be a local extremum. False

c) If x=a is a critical number then f(a) is either a local max or a min. False

d) Not every function has an absolute maximum True

e) Local extrema cannot occur at the end point of a domain True

(3). Find the critical numbers: (2 points each)

You should have a few notes along the way to clarify your work

a) $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 3$

$f'(x) = x^2 - x - 2$

$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

b) $f(x) = x(3-x)^{1/3}$

$f'(x) = (3-x)^{1/3} + \frac{1}{3}x(3-x)^{-2/3}(-1)$

$= (3-x)^{-2/3}(3 - x - \frac{1}{3}x)$

$= (3-x)^{-2/3}(3 - \frac{4}{3}x)$

$= \dfrac{3 - \frac{4}{3}x}{(3-x)^{2/3}} = \dfrac{9-4x}{(3-x)^{2/3}}$

$f'(x) = 0$

$\Rightarrow 9 - 4x = 0$

$x = 9/4$

$f'(x)$ undef.

$\Rightarrow (3-x)^{2/3} = 0$

$x = 3$

Find the absolute extrema of f(x) on the given *interval (Remember, it is implied you will give the output of the function.)*

4) $f(x) = 2\sin x - \cos^2 x;\quad \left[0, \frac{3\pi}{2}\right]$ (3 points)

Find critical #s & $f'(x) = 2\cos x + 2\cos x \sin x$

$= 2\cos x(1 + \sin x)$

$f'(x) = 0 \Rightarrow \cos x = 0$ or $\sin x = -1$

$x = \pi/2, 3\pi/2$

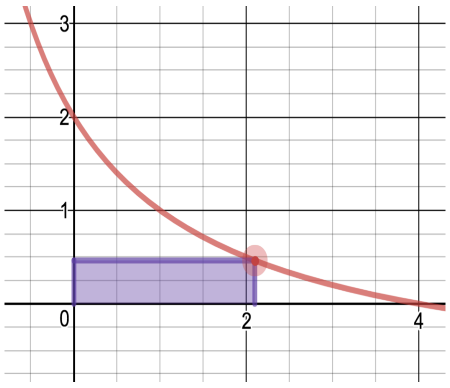
compare f at crit #s & endpoints

x	$\pi/2$	0	$3\pi/2$
$f(x)$	2	-1	-2

Max is 2

min is -2

(5). Find the area of the largest rectangle that can be inscribed by the region bound by the graph of $f(x)=\frac{4-x}{2+x}$ and the coordinate axes in the first quadrant. What is the maximum area? What are the dimensions of the rectangle? (3 points)



$$A = bh = x\left(\frac{4-x}{2+x}\right)$$

$$A = \frac{4x-x^2}{2+x} \quad 0 \le x \le 4$$

Special case – f(x) conts. on closed interval

Find critical #s

$$A'(x) = \frac{(2+x)(4-2x)-(4x-x^2)}{(2+x)^2}$$

$$= \frac{-x^2-4x+8}{(2+x)^2}$$

$A'(x)=0 \Rightarrow -x^2-4x+8=0$

$x = \frac{4 \pm \sqrt{16+32}}{-2}$

$= -2 \pm 2\sqrt{3}$.

$A'(x)$ undefined at $x=-2$

In $[0,4]$

In $[0,4]$, $x=-2+2\sqrt{3}$

x	$-2+2\sqrt{3}$	0	4
$A(x)$	$\underbrace{8-4\sqrt{3}}_{\text{Abs max}}$	0	0

Compare $A(x)$ at crit # and endpoints

Dimensions: $x = -2+2\sqrt{3}$

$y = \frac{4-x}{2+x} \Rightarrow \sqrt{3}-1$

Area $= 8-4\sqrt{3}$